

MATH 147 Review: The Modulus

1. Compute $|z + w|$, $|z|$, and $|w|$ where $z = \sqrt{3} + i$ and $w = 1 + \sqrt{3}i$. Verify that the triangle inequality and the reverse triangle inequality hold.

Solution:

$$\begin{aligned}z + w &= (\sqrt{3} + i) + (1 + \sqrt{3}i) \\&= (1 + \sqrt{3}) + (1 + \sqrt{3})i = (1 + \sqrt{3})\sqrt{2}e^{i\frac{\pi}{4}} \\|z + w| &= (1 + \sqrt{3})\sqrt{2} \approx 3.8637 \\|z| &= |w| = 2 \\||z| - |w|| &= 0 \\|z| + |w| &= 4 \\||z| - |w|| &\leq |z + w| \leq |z| + |w| \checkmark\end{aligned}$$

2. Compute $|z \cdot w|$ where $z = \sqrt{3} + i$ and $w = 1 + \sqrt{3}i$.

Solution:

$$|z \cdot w| = |z| \cdot |w| = 2 \cdot 2 = 4$$